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ANALYTICAL QUANTIFICATION OF STRESSES IN HYPERBOLIC COOLING TOWERS, UNDER ASYMMETRIC WIND LOADING ACTION

QUANTIFICAÇÃO ANALÍTICA DE TENSÕES EM TORRES DE RESFRIAMENTO HIPERBÓLICAS, SOB AÇÃO ASSIMÉTRICA DE CARGA DE VENTO

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ABSTRACT

The objective of this work is to evaluate wind stresses in hyperbolic cooling towers. These non-symmetrical efforts are of utmost importance, since they generally govern the design of such structures. Wind forces, except in cases where the forces generated by the action of earthquakes are considered, produce the main charging case for natural projects of reinforced concrete cooling towers.

KEYWORDS: Hyperbolic. Cooling towers. Earthquakes. Equilibrium equations. Finite Differences

RESUMO

O objetivo deste trabalho é avaliar as tensões do vento em torres de resfriamento hiperbólicas. Esses esforços não simétricos são de extrema importância, visto que geralmente governam o projeto de tais estruturas. O principal caso de carregamento para projetos naturais de torres de resfriamento de concreto armado é produzido por forças do vento, exceto nos casos em que são consideradas as forças geradas pela ação de terremotos.

PALAVRAS-CHAVE: *Hiperbólicas. Torres de resfriamento. Terremotos. Equações de Equilíbrio. Diferenças Finitas*

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1 INTRODUCTION

The objective of this work is to evaluate the wind stresses in hyperbolic cooling towers. These non-symmetrical efforts are of the utmost importance, as they usually govern the design of such structures. The main loading case for natural projects of reinforced concrete cooling towers is produced by forces from the wind, except in cases where forces generated by the action of earthquakes are considered. This study will be based on the "Equations of Equilibrium", and on the respective differential equations, used by Soare (1967), in which "Stress Functions" were used, responsible for the "Asymmetric State of Stress in Hyperbolic Shells of Revolution". For the solution of these differential equations, Soare (1967) employed the "Finite Differences Method"; in this work the same analytic development will be used, however, closed analytic expressions will be obtained for the efforts that govern the asymmetric state of tension in hyperbolic shells of revolution. The importance and appropriateness of the hyperbolic geometric form in terms of economy and strength is well known. There are many proposals for a better understanding of wind action in these special geometric forms of revolution; because the wind constitutes one of the main loading to which these shells can be submitted. An extremely important factor to be considered, together with the incidence of these constructions around the world, is their sensitivity to grouping, that is, their structural behavior in the vicinity of other towers or structures. Soare (1967) uses in its formulation expansions in harmonics (0,1 and 2 for the membrane state) to study the asymmetric action of the wind in these hyperboloids, and an extremely important point in this formulation is that it allows the wind pressure to be applied by the designer independently of its development and resolution, making it quite current in the face of new discoveries and new developments aimed at understanding, properly speaking, the action of wind in cooling towers (or grouping of cooling towers) in the form of hyperbolic shells of revolution. The proposal of this work, looking for a closed, purely analytical method, for the solution of the equations generated by the formulation of Soare (1967) (whose resolution was obtained by finite differences), is summed up literally in the presentation of analytical expressions for the efforts in the hyperbolic shell requested by the wind, for the several harmonics responsible for the membrane state. The search for these analytical expressions requires a consistent mathematical development, always having as a premise the possibility of the designer to freely place the asymmetric loading, ensuring that any new technological proposal to represent the incidence of the wind action in these towers can be implemented integrally, and this is what makes the development of Soare (1967) a current formulation. The results obtained were compared with those available in the publication of Soare (1967), and those obtained through the use of software "MATHEMATICA", which was used to solve the System of Differential Equations.

2 CONSIDERATIONS ON THE TOPIC

Naturally, designed cooling towers are prominent and important structures, especially with regard to their dimensions and relative lightness. According to Armitt (1980), the collapse of three cooling towers at Ferrybridge led to investigations that showed that the effects of wind flow around complex groups of towers and buildings can give rise to stresses, main and floating, much more severe in these shells in the form of towers, than those that appear in an isolated tower, considering the same speed of the wind. It is suspected that these effects due to the grouping were much more severe for the Ferrybridge case than for other groups of cooling towers in the United Kingdom, because there are reports of other towers of similar size, supporting comparable, if not greater, wind speeds.

Therefore, unless the effects due to the grouping of towers are precisely estimated, one encounters the worst case of loading for the design of cooling towers, unless one works with the possibility that the current load will be less severe which is the case of Ferrybridge. Grouping effects more severe than those found in the Ferrybridge investigations are quite rare.

The study involving loads due to wind action should consider the resonance effects that can be quite significant for high average speeds (in high towers) with low damping, and the effect of nearby towers or other structures.

During the construction of these shells substantial deviations between the projected form and the constructed form can occur. These deviations, which are called geometric imperfections, can significantly alter the structural behavior of hyperbolic cooling towers, that is, they can alter static response, free and forced vibration responses, and buckling and post-buckling behavior. Wind forces acting on a cooling tower are in practice non-symmetrical, because of the wind's aerodynamic effects, as well as due to the presence of a group of towers, or any other prominent structures in its vicinity.

Because of their geometry, primarily, these towers resist forces applied through the action of membrane in the plane. The cooling towers are among the largest thin shell structures ever built. The conventional approach for the design of a shell as a cooling tower is based on stresses calculated from elastic analyzes of the structure.

The collapse of cooling towers in the UK at Ferrybridge in 1965 at Ardeer in 1973 and Fiddler's Ferry has attracted the attention of many researchers.

The hyperbolic shape is very efficient to be used in an asymmetric shell. Hyperbolic shells are thin structures, and because of their curvature, are not only aesthetically pleasing, but also exhibit adequate strength.

3 FORMULATION USED

According to Soare (1967), the study of the "Asymmetric State of Stress in Shells", by Membrane Theory, begins by considering the shape of the middle surface.

The equilibrium equations in the φ and θ - coordinates are presented as follows:

$$\frac{\partial \mathrm{N}_{\theta}}{\partial \boldsymbol{\theta}}\ \mathrm{r}_1+\frac{\partial\left(\mathrm{N}_{\varphi\theta}\ \mathrm{r}_0\right)}{\partial \boldsymbol{\varphi}}+\mathrm{N}_{\theta\varphi}\ \mathrm{r}_1\ \cos\boldsymbol{\varphi}+\mathrm{X}\ \mathrm{r}_0\ \mathrm{r}_1=0 \quad (1\ a)$$

$$\frac{\partial \mathrm{N}_{\theta\varphi}}{\partial \boldsymbol{\theta}}\ \mathrm{r}_1+\frac{\partial\left(\mathrm{N}_{\varphi}\ \mathrm{r}_0\right)}{\partial \boldsymbol{\varphi}}-\mathrm{N}_{\theta}\ \mathrm{r}_1\ \cos\boldsymbol{\varphi}+\mathrm{Y}\ \mathrm{r}_0\ \mathrm{r}_1=0 \quad (1\ b)$$

$$\frac{N\varphi}{r_1}+\frac{N_\theta}{r_2}=-Z \quad (1\ c)$$

From (1 c), we have:

$$N_\theta=-Z\ r_2-N_\varphi\ \frac{r_2}{r_1} \quad (2)$$

By including the expression (2) in (1 a) and (1 b), we obtain the following system of partial differential equations that form the basic equations of the problem:

$$-\frac{\partial N_\varphi}{\partial\theta}+\frac{1}{r_2}\ \frac{\partial\left(N_{\varphi\theta}\ r_0\right)}{\partial\varphi}+N_{\theta\varphi}\ \frac{r_1}{r_2}\ \cos\varphi=\left(-X\ \sin\varphi+\frac{\partial Z}{\partial\theta}\right)\ r_1 \quad (3\ a)$$

$$\frac{\partial N_{\theta\varphi}}{\partial\theta}+\frac{1}{r_1}\ \frac{\partial\left(N_\varphi\ r_0\right)}{\partial\varphi}+N_\varphi\ \frac{r_2}{r_1}\ \cos\varphi=-\left(\ Y\ \sin\varphi+Z\ \cos\varphi\right)\ r_2 \quad (3\ b)$$

Adding the differential relation:

$$dz=r_1\ d\varphi\ \sin\varphi, \quad (4)$$

The basic equations can be expressed in the coordinates z and θ as:

$$-\frac{\partial N_\varphi}{\partial\theta}+\frac{\partial\left(N_{\varphi\theta}\ r_0\right)}{\partial z}\ \frac{r_1}{r_2}\ \sin\varphi+N_{\theta\varphi}\ \frac{r_1}{r_2}\ \cos\varphi=\left(-X\ \sin\varphi+\frac{\partial Z}{\partial\theta}\right)\ r_1 \quad (5\ a)$$

$$\frac{\partial N_{\theta\varphi}}{\partial\theta}+\frac{\partial\left(N_\varphi\ r_0\right)}{\partial z}\ \sin\varphi+N_\varphi\ \frac{r_2}{r_1}\ \cos\varphi=-\left(Y\ \sin\varphi+Z\ \cos\varphi\right)\ r_2 \quad (5\ b)$$

There are two integration methods for systems (3) and (5), ie: a) use of trigonometric series to express the loads and forces, through which, the problem of integrating a system of partial differential equations for the integration of a system of ordinary differential equations is reduced; b) direct integration of the system of partial differential equations (3) and (5). In both methods, the problem can still be simplified by the introduction of a stress function F. Equations (3) can be written in the form:

$$\frac{\partial\left(N_{\varphi\theta}\ r_0^2\right)}{\partial\varphi}-\frac{r_0}{\sin^2\varphi}\ \frac{\partial\left(N_\varphi\ r_0\ \sin\varphi\right)}{\partial\theta}=\left(-X\ \sin\varphi+\frac{\partial Z}{\partial\theta}\right)\ r_0\ r_1\ r_2 \quad (6\ a)$$

$$\frac{\partial(N_\varphi \; r_0 \; \sin\varphi)}{\partial\varphi} + \frac{r_1 \; \sin\varphi}{r_0^2} \; \frac{\partial(N_{\varphi\theta} \; r_0^2)}{\partial\theta} = -(Y \; \sin\varphi + Z \; \cos\varphi) \; r_0 \; r_1 \tag{6 b}$$

It is verified that with the use of a tension function F, such that the forces are represented by means of the relations:

$$\left.\begin{aligned} N_{\varphi\theta} &= \frac{1}{r_0^2} \; \frac{\partial F}{\partial\theta} \;, \\ N_\varphi &= \frac{\sin\varphi}{r_0^2} \; \frac{\partial F}{\partial\varphi} + r_1 \; \sin\varphi \; \int X \; d\theta - Z \; r_1 \;, \\ N_\theta &= -\frac{1}{r_0 \; r_1} \; \frac{\partial F}{\partial\varphi} - r_0 \; \int X \; d\theta \;, \end{aligned}\right\} \tag{7}$$

Equation (6a) is identically satisfied, and (6b) becomes:

$$\frac{\partial}{\partial\theta}\left(\frac{\sin^2\varphi}{r_0} \; \frac{\partial F}{\partial\varphi}\right) + \frac{r_1}{r_0^2} \; \sin\varphi \; \frac{\partial^2 F}{\partial\theta^2} = -(Y \; \sin\varphi + Z \; \cos\varphi) \; r_0 \; r_1$$
$$-\frac{\partial}{\partial\varphi}\left[r_1 \; \sin^2\varphi \; \left(r_0 \; \int X \; d\theta - Z \; r_2 \right) \right] \tag{8}$$

Since the stress function F is determined by the integration of equation (8), the forces in the shell are deduced from (7).

A second stress function can be introduced by taking:

$$\left.\begin{aligned} N_{\varphi\theta} &= -\frac{1}{r_1 \; \sin\varphi} \; \frac{\partial F}{\partial\varphi} - r_0 \; \int Y \; d\theta - r_2 \; \cos\varphi \; \int Z \; d\theta, \\ N_\varphi &= \frac{1}{r_0 \; \sin\varphi} \; \frac{\partial F}{\partial\theta} \;, \\ N_\theta &= -\frac{1}{r_1 \; \sin^2\varphi} \; \frac{\partial F}{\partial\theta} - Z \; r_2 \;. \end{aligned}\right\} \tag{9}$$

Thus, equation (6b) is identically satisfied, and (6a) becomes:

$$\frac{\partial}{\partial\varphi}\left(\frac{r_0^2}{r_1 \; \sin\varphi} \; \frac{\partial F}{\partial\varphi}\right) + \frac{r_0}{\sin^2\varphi} \; \frac{\partial^2 F}{\partial\theta^2} = -\left(-X \; \sin\varphi + \frac{\partial Z}{\partial\theta}\right) \; r_0 \; r_1 \; r_2$$
$$-\frac{\partial}{\partial\boldsymbol{\varphi}}\left(\mathrm{r}_0^3 \int \mathrm{Y} \; \mathrm{d}\boldsymbol{\theta} + \mathrm{r}_0^3 \; \mathrm{cot\,g}\boldsymbol{\varphi} \int \mathrm{Z} \; \mathrm{d}\boldsymbol{\theta}\right) \tag{10}$$

For the complete determination of the forces in the shell, knowledge of the initial values at the top is necessary. For open shells, of the conditions of static equilibrium, we have in the upper parallel circle that:

$$\mathrm{N}_{\varphi\theta} = \mathrm{N}_{\varphi} \equiv 0 \tag{11}$$

A common procedure for integrating the systems of differential equations (3) and (5), or for equations (8) and (10) is the development of expressions, for applied loads and internal forces components, in series trigonometric in terms of the variable θ. Such expansions in series are of the form:

$$\left.\begin{aligned} X &= \sum X_n \ \sin(n\theta) \\ Y &= \sum Y_n \ \cos(n\theta) \\ Z &= \sum Z_n \ \cos(n\theta) \end{aligned}\right\} (n = 0 \ , 1 \ , \ 2 \ ,...) \tag{12}$$

$$\left.\begin{aligned} N_\varphi &= \sum N_\varphi^{(n)} = \sum N_{\varphi n} \ \cos(n\theta) \\ N_\theta &= \sum N_\theta^{(n)} = \sum N_{\theta n} \ \cos(n\theta) \\ N_{\varphi\theta} &= \sum N_{\varphi\theta}^{(n)} = \sum N_{\varphi\theta n} \ \sin(n\theta) \end{aligned}\right\} (n = 0 \ , 1 \ , \ 2 \ ,...), \tag{13}$$

By introducing the expressions for Z , N_φ *and* N_θ of (12) and (13) into (2), and dividing by $\cos(n\theta)$, one obtains:

$$N_{\theta n} = -Z_n \ r_2 - N_{\varphi n} \ \frac{r_2}{r_1} \tag{14}$$

By introducing the trigonometric expressions in the system of equations (3), and dividing by $\sin(n\theta) \ e \ \cos(n\theta)$, respectively, we get:

$$n \ r_2 \ N_{\varphi n} + \frac{d(N_{\varphi\theta n} \ r_0)}{d\varphi} + N_{\theta\varphi n} \ r_1 \ \cos\varphi = -\left(X_n \ \sin\varphi + n \ Z_n\right) r_1 \ r_2 \tag{15 a}$$

$$n \ r_1 \ N_{\theta\varphi n} + \frac{d(N_{\varphi n} \ r_0)}{d\varphi} + N_{\varphi n} \ r_2 \ \cos\varphi = -\left(Y_n \ \sin\varphi + Z_n \ \cos\varphi\right) r_1 \ r_2 \tag{15 b}$$

Equations (15) may also be written in other forms, which may be more convenient for resolution. With the computation of the derivatives of the forces, one obtains:

$$\left.\begin{aligned} &\frac{dN_{\varphi\theta n}}{d\varphi} + \frac{2}{r_0}\frac{dr_0}{d\varphi} \ N_{\theta\varphi n} + \frac{n}{\sin\varphi} \ N_{\varphi n} = -\left(X_n + \frac{n}{\sin\varphi} \ Z_n\right) r_1 \ , \\ &\frac{dN_{\varphi n}}{d\varphi} + \left(\frac{1}{r_0}\frac{dr_0}{d\varphi} + \cot g\varphi\right) N_{\varphi n} + \frac{n \ r_1}{r_0} \ N_{\varphi\theta n} = -\left(Y_n + Z_n \ \cot g\varphi\right) r_1 \ . \end{aligned}\right\} \tag{16}$$

From the system (6) we deduce:

$$\left.\begin{aligned} &\frac{d\left(N_{\varphi\theta n}\ r_0^2\right)}{d\varphi}+\frac{n\ r_2}{\sin\varphi}\left(N_{\varphi n}\ r_0\ \sin\varphi\right)=-\left(X_n\ r_0+n\ Z_n\ r_2\right)\ r_0\ r_1\ , \\ &\frac{d\left(N_{\varphi n}\ r_0\ \sin\varphi\right)}{d\varphi}+\frac{n\ r_1}{r_0\ r_2}\left(N_{\varphi\theta n}\ r_0^2\right)=-\left(Y_n\ \sin\varphi+Z_n\ \cos\varphi\right)\ r_0\ r_1\ . \end{aligned}\right\} \tag{17}$$

This system can be used directly, taking the magnitudes $N_{\varphi\theta n}\ r_0^2\ e\ N_{\varphi n}\ r_0\ \sin\varphi$ as new unknowns:

$$F_1=-\frac{1}{n}\ N_{\varphi\theta}\ r_0^2\ \text{ e }\ F_2=N_{\varphi}\ r_0\ \text{sen}\,\boldsymbol{\varphi} \tag{18}$$

According to Soare (1967), the determination of the pressure variation and wind suction along a parallel circle, in structures of circular cross-section, is a very complex problem due to the large number of parameters involved. One of these, considered essential, is the nature of the surface, ie, smooth or rough surface.

In Soare (1967), the hyperbolic tower required by the wind, whose efforts are determined by the finite difference method, is presented in Figure (1).

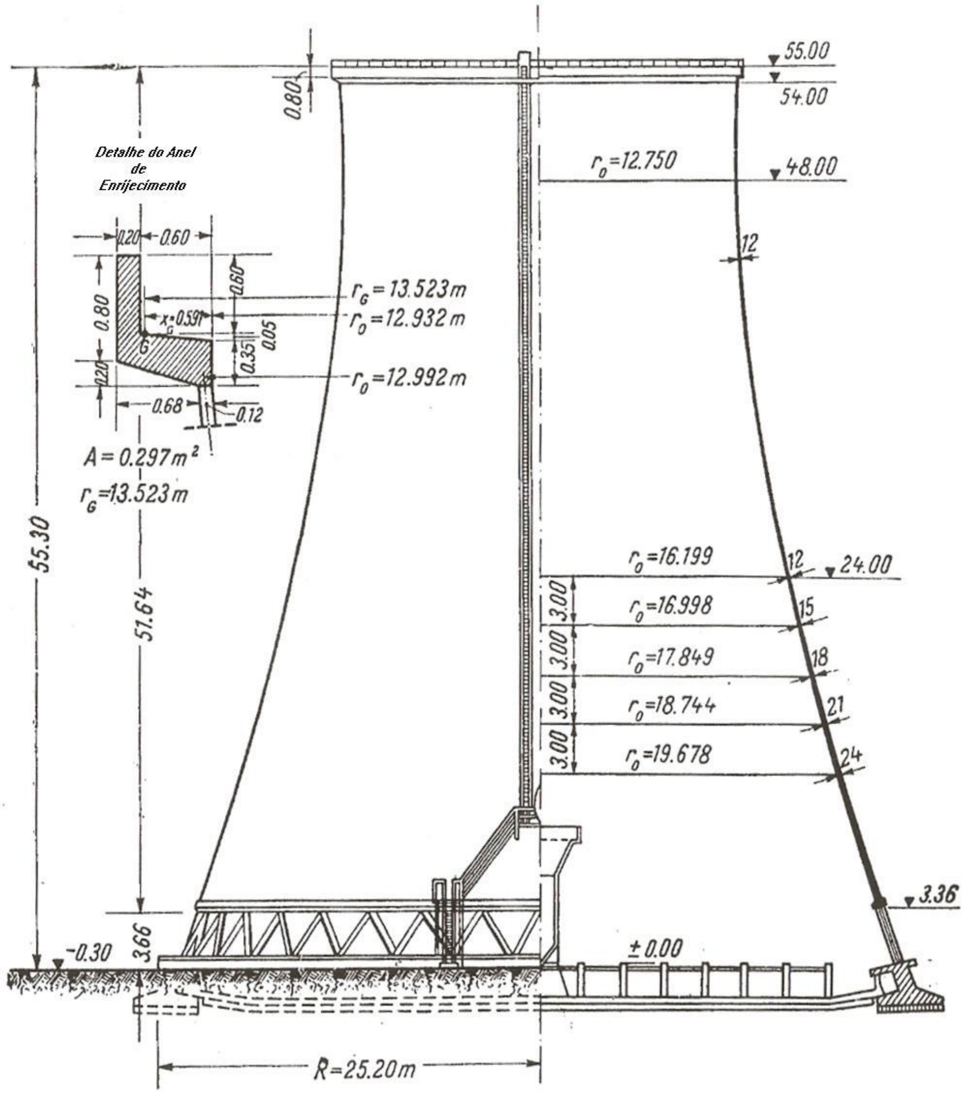

Figure (1): Hyperbolic cooling tower. (Source: Soare, 1967, p. 100)

Assuming that the variation of the wind pressure is sinusoidal along the meridian (one cannot take into account the variation of the wind pressure with the height of the tower, since in practice, $\sin\varphi$ is insignificant compared to z), the components of the applied loads are written as:

$$\left.\begin{aligned} X &= Y \equiv 0 \\ Z &\approx p_W \sin\varphi \end{aligned}\right\} \qquad (19)$$

According to Soares (1967), for the State of Membrane Tension, harmonics are limited in $n = 0\ ,\ 1\ ,\ 2$.

The harmonic $n = 0$ corresponds to the symmetrical state of tension. The results are easily obtained by direct integration. For the other harmonics the stress functions, $F_1 \ e \ F_2$ defined in (18), are used. Using the stress functions $F_1 \ e \ F_2$ defined in (18), and the relation (4) in the relations (17), these can be written in terms of the variable z:

$$\left.\begin{aligned} \frac{dF_1}{dz} - \frac{r_0}{r_1 \ \sin^3 \varphi} \ F_2 &= r_2^2 \left(Z_n + \frac{1}{n} \ X_n \ \sin\varphi \right) \ , \\ \frac{dF_2}{dz} - \frac{n^2}{r_0^2} \ F_1 &= -r_2 \left(Y_n \ \sin\varphi + Z_n \ \cos\varphi \right) . \end{aligned}\right\} \tag{20}$$

In the particular case of the hyperboloid of revolution, there is a relation between the geometric elements, that is,

$$-\frac{\mathrm{r}_0}{\mathrm{r}_1 \ \mathrm{sen}^3 \boldsymbol{\varphi}} = +\frac{\mathrm{a}^2}{\mathrm{r}_0^2 \ \mathrm{tg}^2 \mathrm{A}} \tag{21}$$

Which can be easily confirmed in the consideration of the expressions for $r_0 \ e \ r_1$:

$$\mathrm{r}_0 = \frac{\mathrm{a} \ \cos \mathrm{A} \ \mathrm{sen}\, \boldsymbol{\varphi}}{\sqrt{\cos^2 \mathrm{A} - \cos^2 \boldsymbol{\varphi}}} \tag{22}$$

$$\mathrm{r}_1 = -\frac{\mathrm{a} \ \mathrm{sen}^2 \mathrm{A} \ \cos \mathrm{A}}{\left(\cos^2 \mathrm{A} - \cos^2 \boldsymbol{\varphi}\right)^{\frac{3}{2}}} \tag{23}$$

For the particular case, where the loading components act $\left(X_n = Y_n \equiv 0 \ , \ Z_n = p_w \ \sin\varphi\right)$, the equations (20) can be written in a simpler way, that is,

$$\left.\begin{aligned} \frac{dF_1}{dz} + \frac{a^2}{r_0^2 \ tg^2 A} \ F_2 &= \frac{p_w \ r_0^2}{\sin\varphi} \ , \\ \frac{dF_2}{dz} - \frac{n^2}{r_0^2} \ F_1 &= -p_w \ r_0 \ \cos\varphi \ . \end{aligned}\right\} \tag{24}$$

Eliminating F_2 of the two relations, and doing so $F_1 = F$, we obtain the equation:

$$\frac{\mathrm{d}^2\mathrm{F}}{\mathrm{dz}^2} + \frac{2 \ \cot\mathrm{g}\boldsymbol{\varphi}}{\mathrm{r}_0} \frac{\mathrm{dF}}{\mathrm{dz}} + \frac{\mathrm{n}^2 \ \mathrm{a}^2}{\mathrm{r}_0^4 \ \mathrm{tg}^2\mathrm{A}} \mathrm{F} = \frac{4 \ \mathrm{p_w} \ \cos\boldsymbol{\varphi}}{\mathrm{r}_0} \left(\mathrm{r}_2^2 + \frac{\mathrm{a}^2}{2 \ \mathrm{tg}^2\mathrm{A}} \right) \tag{25}$$

Which will be responsible for the study involving the analysis of tensions caused by the wind action in the tower of cooling, in the form of hyperboloid of revolution.

Comments:

1) $r_0 = a\sqrt{1+\left(\frac{z}{b}\right)^2}$ = radius of the parallel circle of the tower, where:

$tgA = \frac{b}{a}$ (A is the angle formed by the meridian asymptotic of the hyperbola and the axis $O\ r_0$);

$z =$ vertical axis of the tower;

2) $r_1\ e\ r_2$ are the principal radii of curvature;

3) X , $Y\ e\ Z$ are the external loads of the shell, being Z the component represented by the wind action, that is, the component orthogonal to the tangents of the tower;

4) $tg\varphi = \frac{dz}{dr_0} = \pm\frac{b}{a}\frac{r_0}{\sqrt{r_0^2-a^2}} = \frac{r_0}{z}\ tg^2A$;

5) Cartesian equation of the hyperbola meridian: $\frac{r_0^2}{a^2}-\frac{z^2}{b^2}=1$.

4 METHODOLOGY USED

The study of the stresses in a hyperbolic tower will be initiated by assuming the membrane state for the structure. In these conditions, according to Soare (1967), we will analyze the equations (24) and (25), in which the harmonics 0, 1, and 2 will be focused.

Taking the homogeneous part of equation (25), we have:

$$\frac{d^2F}{dz^2}+\frac{2\ Cot\varphi}{r_0}\frac{dF}{dz}+\frac{n^2\ a^2}{r_0^4\ tg^2A}\ F=0 \tag{26}$$

Whose solution, according to Kamke (1983), is

$$\phi=\phi_1=C\ \cos\left[n\ arc\ tg\left(\frac{z}{b}\right)\right] \tag{27}$$

And

$$\phi_2=-\frac{d\phi_1}{dz}\frac{r_0^2\ tg^2A}{a^2}=C\ n\ \sin\left[n\ arc\ tg\left(\frac{z}{b}\right)\right]\frac{b}{a^2} \tag{28}$$

The Wronskian, according to Kamke (1983), is given by:

$$W=\phi_1\ \phi_2^{'}-\phi_1^{'}\ \phi_2=C^2\ n^2\ \frac{tg^2A}{(b^2+z^2)} \tag{29}$$

4.1 STUDY FOR THE HARMONIC "0" ($n=0$)

For harmonic 0, $n=0$, corresponding to the symmetric state of stress, equations (24) can be solved by direct integration:

$$\frac{dF_2}{dz} = \frac{dF_2}{dr_0}\ \frac{dr_0}{dz} = \frac{dF_2}{dr_0}\ \frac{1}{tg\varphi} = -p_w\ r_0\ \cos\varphi \quad \Rightarrow \quad \frac{dF_2}{dr_0} = -p_w\ r_0\ \cos\varphi\ tg\varphi \quad \Rightarrow$$

$$F_2 = -p_w\ tg\ A \int \frac{r_0^2}{\sqrt{\sec^2 A\ r_0^2 - a^2}}\ dr_0 \quad \Rightarrow F_2 = -\frac{p_w\ \sin A\ \cos A}{2}\ .$$

$$.\left\{ r_0\sqrt{\sec^2 A\ r_0^2 - a^2} + \frac{a^2}{\sec A}\ \ln\left[2\ \sec A\sqrt{\sec^2 A\ r_0^2 - a^2} + 2\ \sec^2 A\ r_0\right]\right\} - p_w\ tgA\ C \qquad (30)$$

Where the constant C is determined by the initial condition, which, according to Soare (1967), imposes, at the top of the hyperbolic tower, that the stresses due to wind incidence are equal to zero (open top of the hyperbolic tower).

And,

$$N_{\varphi 0} = \frac{1}{r_0\ \sin\varphi}\ F_2 \qquad (31)$$

Or how $N_{\varphi 0} = f(z)$:

$$N_{\varphi 0} = -\ p_w \frac{\sqrt{b^2\sin^2 A + z^2}}{\left(b^2 + z^2\right)} \left\{ \frac{\sqrt{b^2 + z^2}\ \sqrt{b^2\sin^2 A + z^2}}{2\ tgA} + \frac{tgA}{\cos A}\ C\ + \right.$$

$$\left. + \frac{b^2\ \cos^2 A}{2\ tg\ A}\ \ln\left(\frac{2}{\cos\ A\ \sin\ A}\left(\sqrt{b^2\sin^2 A + z^2} + \sqrt{b^2 + z^2}\right)\right)\right\} \qquad (32)$$

4.2 STUDY FOR THE HARMONIC "1" ($n = 1$)

For harmonic 1, $n = 1$, equations (27), (28) and (29) respectively become:

$$\phi_1 = C\ \cos\left[1\ arc\ tg\left(\frac{z}{b}\right)\right] = C\ \cos\left[1\ arc\ \cos\left(\frac{b}{\sqrt{b^2 + z^2}}\right)\right] = C\ \frac{b}{\sqrt{b^2 + z^2}} \qquad (33)$$

$$\phi_2 = C\ 1\ \sin\left[1\ arc\ tg\left(\frac{z}{b}\right)\right]\ \frac{b}{a^2} = C\ \sin\left[arc\ \sin\left(\frac{z}{\sqrt{b^2 + z^2}}\right)\right]\ \frac{b}{a^2} = C\ \frac{b\ z}{a^2\sqrt{b^2 + z^2}} \qquad (34)$$

$$W = C^2\ 1^2\ \frac{tg^2 A}{(b^2 + z^2)} = C^2\ \frac{tg^2 A}{(b^2 + z^2)} \qquad (35)$$

According to Kamke (1983), the complete solution for equation (25) is given by:

$$F = F_1 = \phi_2 \int \frac{\phi_1\ h}{W} dz - \phi_1 \int \frac{\phi_2\ h}{W} dz + C_1\ \phi_1 + C_2\ \phi_2 \quad \Rightarrow$$

$$\Rightarrow F_1 = \frac{b\ z}{a^2\sqrt{b^2 + z^2}} \int \left[4\ p_w \left(\frac{a^2}{2} + b^2\right)\ b\ \cot g^4 A\ \sin\ A\ \frac{z}{\sqrt{b^2\sin^2 A + z^2}} + \right.$$

$$+4\ p_w\ b\ \cot g^4 A\ \cos ec\ A\ \frac{z^3}{\sqrt{b^2\sin^2 A+z^2}}\Bigg]\ dz+$$

$$-\frac{b}{\sqrt{b^2+z^2}}\int\Bigg[4\ p_w\left(\frac{a^2}{2}+b^2\right)\frac{1}{b}\ \cot g^2 A\ \sin A\ \frac{z^2}{\sqrt{b^2 sen^2 A+z^2}}+$$

$$+4\ p_w\ \frac{1}{b}\ \cot g^2 A\ \cos ec\ A\ \frac{z^4}{\sqrt{b^2 sen^2 A+z^2}}\Bigg]\ dz+C_1\ \frac{b}{\sqrt{b^2+z^2}}+C_2\ \frac{b\ z}{a^2\ \sqrt{b^2+z^2}} \qquad (36)$$

Where C_1 and C_2 are constants determined through the initial condition that, according to Soare (1967), imposes, at the top of the hyperbolic tower, that the stresses due to the wind incidence are equal to zero (open top of the hyperbolic tower).

$$F_1=\frac{1}{12\ a^2\sqrt{b^2+z^2}}\Bigg(\ p_w\ z\ \sqrt{b^2\sin^2 A+z^2}\ \left(-6a^4+8b^4+3a^2\ (3b^2-4z^2)\ +\right.$$

$$\left.+\left(6a^4+15a^2b^2+8b^4\right)\ \cos(2A)\right)\ \cot^2 A\ \cos ec\ A+16\ b^2 p_w\ z^3\sqrt{b^2\sin^2 A+z^2}\ \cot^4 A\ \cos ec\ A+$$

$$+6\ b\ \Big(\ 2\ \Big(\ a^2\left(-9660{,}47\ p_w\right)+\left(-12010{,}30\ p_w\right)\ z\ \Big)+$$

$$+a^2\ b\ \left(2\ a^2+b^2\right)\ p_w\cos^2 A\ \ln\left(\ 2\ z+2\ \sqrt{b^2\sin^2 A+z^2}\right)\ \sin A\ \Big)\Bigg) \qquad (37)$$

$$N_{\varphi\,\theta_1}=-\frac{tg^2 A}{\left(b^2+z^2\right)}\ F_1 \qquad (38)$$

$$N_{\varphi_1}=\frac{\sqrt{b^2\sin^2 A+z^2}}{a^2\cos A}\left[\frac{p_w\ \left(b^2+z^2\right)}{tg^2 A}\ \frac{\sqrt{b^2\sin^2 A+z^2}}{\sin A\ \sqrt{b^2+z^2}}-\frac{d\ F_1}{dz}\right] \qquad (39)$$

4.3 STUDY FOR THE HARMONIC "2"($n=2$)

For harmonic 2, the equations (27), (28) and (29) respectively become:

$$\phi=\phi_1=C\ \cos\left[2\ arc\ tg\left(\frac{z}{b}\right)\right]=C\ \left\{\cos^2\left[arc\ tg\left(\frac{z}{b}\right)\right]-\sin^2\left[arc\ tg\left(\frac{z}{b}\right)\right]\right\}=$$

$$=C\ \left\{\cos^2\left[arc\ \cos\left(\frac{b}{\sqrt{b^2+z^2}}\right)\right]-\sin^2\left[arc\ \sin\left(\frac{z}{\sqrt{b^2+z^2}}\right)\right]\right\}=C\ \frac{b^2}{\left(b^2+z^2\right)}-\frac{z^2}{\left(b^2+z^2\right)}\Rightarrow$$

$$\Rightarrow\phi_1=C\ \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)} \qquad (40)$$

$$\phi_2=-\frac{d\phi_1}{dz}\ \frac{r_0^2\ tg^2 A}{a^2}=C\ 2\ \sin\left[2\ arc\ tg\left(\frac{z}{b}\right)\right]\ \frac{b}{a^2}=$$

$$= C\ 2\ 2\ \sin\left[arc\ tg\left(\frac{z}{b}\right)\right]\ \cos\left[arc\ tg\left(\frac{z}{b}\right)\right]\ \frac{b}{a^2} =$$

$$= C\ 4\ \sin\left[arc\ \sin\left(\frac{z}{\sqrt{b^2+z^2}}\right)\right]\ \cos\left[arc\ \cos\left(\frac{b}{\sqrt{b^2+z^2}}\right)\right]\ \frac{b}{a^2} = C\ 4\ \frac{z}{\sqrt{b^2+z^2}}\ \frac{b}{\sqrt{b^2+z^2}}\ \frac{b}{a^2} \Rightarrow$$

$$\Rightarrow \phi_2 = C\ 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \qquad (41)$$

$$W = C^2\ 2^2\ \frac{tg^2 A}{(b^2+z^2)} \Rightarrow W = C^2\ 4\ \frac{tg^2 A}{(b^2+z^2)} \qquad (42)$$

According to Kamke (1983), the complete solution for equation (25) is given by:

$$F = F_1 = \phi_2 \int \frac{\phi_1\ h}{W} dz - \phi_1 \int \frac{\phi_2\ h}{W} dz + C_1\ \phi_1 + C_2\ \phi_2\ \Rightarrow$$

$$\Rightarrow F_1 = 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \int \left[p_w \left(\frac{a^2}{2} + b^2\right)\ \cot g^4 A\ \sin\ A\ \frac{\left(b^2-z^2\right)\ z}{\sqrt{b^2+z^2}\ \sqrt{b^2 sen^2 A + z^2}} + \right.$$

$$\left. + p_w \cot g^4 A\ \cos ec\ A\ \frac{\left(b^2-z^2\right)\ z^3}{\sqrt{b^2+z^2}\ \sqrt{b^2 sen^2 A + z^2}} \right] dz\ -\ \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)}$$

$$\int \left[4\ p_w \left(\frac{a^2}{2} + b^2\right)\ \cot g^2 A\ \sin\ A\ \frac{z^2}{\sqrt{b^2+z^2}\ \sqrt{b^2 sen^2 A + z^2}}\ +\ 4\ p_w\ \cot g^2 A\ \cos ec\ A \right.$$

$$\left. \frac{z^4}{\sqrt{b^2+z^2}\ \sqrt{b^2 sen^2 A + z^2}} \right] dz + C_1 \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)} + C_2\ 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \qquad (43)$$

In equation (43), which is responsible for determining the expression of F_1 for harmonic 2 ($n = 2$), only the first of the two integrals has an analytical solution; the second, because it is an elliptic integral, is only solved numerically, that is, it is not possible to solve this integral by presenting an analytical expression as a result; with the inclusion of integration limits, the numerical solution of this integral will be "a numerical value".

By establishing a certain preparation of this elliptic integral, it becomes possible the analytical resolution of this by employing the Heron Formula to approximate the square root of an integer that is not a perfect square. The use of Heron's formula allows the presentation of a closed analytic expression for the stress function F_1 responsible for harmonic 2 ($n = 2$).

In spite of the first integral, in equation (43), to have analytical solution, for reasons of simplification, in both integrals in this equation will be used the Heron Formula. An analytical resolution to the stresses due to the wind that are incident on a hyperbolic cooling tower can greatly subsidize the structural design of these towers, greatly increasing the resources available to the designers.

For the use of the Heron formula in both integrals, a prior preparation in the expression of F_1, responsible for harmonic 2 ($n = 2$), in equation (43) is required, as shown in equation (44) below:

$$F = F_1 = \phi_2 \int \frac{\phi_1\ h}{W} dz - \phi_1 \int \frac{\phi_2\ h}{W} dz + C_1\ \phi_1 + C_2\ \phi_2 \ \Rightarrow$$

$$\Rightarrow F_1 = 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \int \Bigg[p_w \left(\frac{a^2}{2}+b^2\right) \cot g^4 A\ \sin\ A\ \frac{\left(b^2-z^2\right) z}{(b^2 \sin^2 A + z^2)} \frac{\sqrt{b^2 \sin^2 A + z^2}}{\sqrt{b^2+z^2}} +$$

$$+ p_w \cot g^4 A\ \cos ec\ A\ \frac{\left(b^2-z^2\right) z^3}{(b^2 \sin^2 A + z^2)} \frac{\sqrt{b^2 \sin^2 A + z^2}}{\sqrt{b^2+z^2}} \Bigg] dz\ -\ \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)}$$

$$\int \Bigg[4\ p_w \left(\frac{a^2}{2}+b^2\right) \cot g^2 A\ \sin\ A\ \frac{z^2}{(b^2 \sin^2 A + z^2)} \frac{\sqrt{b^2 \sin^2 A + z^2}}{\sqrt{b^2+z^2}}\ +\ 4\ p_w\ \cot g^2 A\ \cos ec\ A$$

$$\frac{z^4}{(b^2 \sin^2 A + z^2)} \frac{\sqrt{b^2 \sin^2 A + z^2}}{\sqrt{b^2+z^2}} \Bigg] dz + C_1\ \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)} + C_2\ 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \qquad (44)$$

Doing:

$$\frac{\sqrt{b^2 sen^2 A + z^2}}{\sqrt{b^2+z^2}} = \frac{\left(\frac{2\sqrt{b^2+z^2}(z^2+b^2\mathrm{Sin}[A]^2)(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}+\frac{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}{8\sqrt{b^2+z^2}(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}\right)}{\sqrt{b^2+z^2}} \qquad (45)$$

The expression of F_1 responsible for harmonic 2 ($n = 2$), becomes (with the substitution of C_1 and C_2 by JJJ and KKK, respectively):

$$F = F_1 = \phi_2 \int \frac{\phi_1\ h}{W} dz - \phi_1 \int \frac{\phi_2\ h}{W} dz + C_1\ \phi_1 + C_2\ \phi_2 \ \Rightarrow$$

$$\Rightarrow F_1 = 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \int \Bigg[p_w \left(\frac{a^2}{2}+b^2\right) \cot g^4 A\ \sin\ A\ \frac{\left(b^2-z^2\right) z}{(b^2 \sin^2 A + z^2)}$$

$$\frac{\left(\frac{2\sqrt{b^2+z^2}(z^2+b^2\mathrm{Sin}[A]^2)(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}+\frac{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}{8\sqrt{b^2+z^2}(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}\right)}{\sqrt{b^2+z^2}} +$$

$$+ p_w \cot g^4 A\ \cos ec\ A\ \frac{\left(b^2-z^2\right) z^3}{(b^2 \sin^2 A + z^2)}$$

$$\frac{\left(\frac{2\sqrt{b^2+z^2}(z^2+b^2\mathrm{Sin}[A]^2)(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}+\frac{4(b^2+z^2)(z^2+b^2\mathrm{Sin}[A]^2)+(b^2+2z^2+b^2\mathrm{Sin}[A]^2)^2}{8\sqrt{b^2+z^2}(b^2+2z^2+b^2\mathrm{Sin}[A]^2)}\right)}{\sqrt{b^2+z^2}} \Bigg] dz\ -$$

$$
-\frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)}\int\left[4\ p_w\left(\frac{a^2}{2}+b^2\right)\ \cot g^2 A\ \ \sin\ A\ \frac{z^2}{(b^2\sin^2 A+z^2)}\right.
$$

$$
\frac{\left(\frac{2\sqrt{b^2+z^2}(z^2+b^2\text{Sin}[A]^2)(b^2+2z^2+b^2\text{Sin}[A]^2)}{4(b^2+z^2)(z^2+b^2\text{Sin}[A]^2)+(b^2+2z^2+b^2\text{Sin}[A]^2)^2}+\frac{4(b^2+z^2)(z^2+b^2\text{Sin}[A]^2)+(b^2+2z^2+b^2\text{Sin}[A]^2)^2}{8\sqrt{b^2+z^2}(b^2+2z^2+b^2\text{Sin}[A]^2)}\right)}{\sqrt{b^2+z^2}}+
$$

$$
+4\ p_w\ \cot g^2 A\ \ \cos ec\ A\ \frac{z^4}{(b^2\sin^2 A+z^2)}
$$

$$
\left.\frac{\left(\frac{2\sqrt{b^2+z^2}(z^2+b^2\text{Sin}[A]^2)(b^2+2z^2+b^2\text{Sin}[A]^2)}{4(b^2+z^2)(z^2+b^2\text{Sin}[A]^2)+(b^2+2z^2+b^2\text{Sin}[A]^2)^2}+\frac{4(b^2+z^2)(z^2+b^2\text{Sin}[A]^2)+(b^2+2z^2+b^2\text{Sin}[A]^2)^2}{8\sqrt{b^2+z^2}(b^2+2z^2+b^2\text{Sin}[A]^2)}\right)}{\sqrt{b^2+z^2}}\right]dz\ +
$$

$$
+JJJ\ \frac{\left(b^2-z^2\right)}{\left(b^2+z^2\right)}+KKK\ \ 4\ tg^2 A\ \frac{z}{\left(b^2+z^2\right)} \tag{46}
$$

Then, the analytic expression of F_1, achieved with the resolution of equation (46), is given by:

$$
F_1=\phi_2\int\frac{\phi_1\ h}{W}dz-\phi_1\int\frac{\phi_2\ h}{W}dz+JJJ\ \ \phi_1+KKK\ \ \phi_2\ \ \Rightarrow F_1=
$$

$$
\frac{1}{256(b^2+z^2)}(256\text{JJJ}(b^2-z^2)+
$$

$$
\text{PwzCsc}[A](-256z^4\text{Cot}[A]^2+128z^2(-a^2+5b^2+(a^2+b^2)\text{Cos}[2A])\text{Cot}[A]^2
$$

$$
-32b^2(-a^2+2b^2+(a^2+2b^2)\text{Cos}[2A])\text{Cot}[A]^2\text{Log}[b^2+z^2]
$$

$$
+\sqrt{2}b^2(2a^2+30\sqrt{2}a^2-32b^2-29\sqrt{2}b^2-4(8\sqrt{2}a^2
$$

$$
+(8+7\sqrt{2})b^2)\text{Cos}[2A]+(2(-1+\sqrt{2})a^2+\sqrt{2}b^2)\text{Cos}[4A])
$$

$$
\text{Cot}[A]^2\text{Log}[6b^2+8z^2+2\sqrt{2}b^2\text{Cos}[A]^2-2b^2\text{Cos}[2A]]-16a^2b^2\text{Cos}[A]^2
$$

$$
(-3+\text{Cos}[2A])\text{Log}[b^2+2z^2-b^2\text{Cos}[2A]]+8b^2(-7+\text{Cos}[2A])
$$

$$
(-a^2+b^2+(a^2+b^2)\text{Cos}[2A])\text{Cot}[A]^2\text{Log}[3b^2+4z^2-b^2\text{Cos}[2A]]+
$$

$$
\sqrt{2}b^2(-2a^2+30\sqrt{2}a^2+32b^2-29\sqrt{2}b^2-4(8\sqrt{2}a^2+
$$

$$
(-8+7\sqrt{2})b^2)\text{Cos}[2A]+(2(1+\sqrt{2})a^2+\sqrt{2}b^2)\text{Cos}[4A])\text{Cot}[A]^2
$$

$$
\text{Log}[2(-3b^2-4z^2+\sqrt{2}b^2\text{Cos}[A]^2+b^2\text{Cos}[2A])])-
$$

$$
64\text{Pw}(b^2-z^2)\text{Csc}[A](\frac{16}{3}z^3\text{Cot}[A]^2-4z(-a^2+b^2+(a^2+b^2)\text{Cos}[2A])\text{Cot}[A]^2
$$

$$
+\frac{1}{2}\text{ArcTan}[\frac{2z}{\sqrt{-b^2(-3+\text{Cos}[2A])}}]
$$

$$
\sqrt{-b^2(-3+\text{Cos}[2A])}(-a^2+b^2+(a^2+b^2)\text{Cos}[2A])\text{Cot}[A]^2+
$$

$$
\frac{1}{2}b\text{ArcTan}[\frac{z}{b}](-a^2+2b^2+(a^2+2b^2)\text{Cos}[2A])\text{Cot}[A]^2
$$

$$
+\frac{b\text{ArcTan}[\frac{2\sqrt{2}z}{b\sqrt{6+\sqrt{2}+(-2+\sqrt{2})\text{Cos}[2A]}}]}{8\sqrt{6+\sqrt{2}+(-2+\sqrt{2})\text{Cos}[2A]}}
$$

$$
(-2a^2-14\sqrt{2}a^2+16b^2+13\sqrt{2}b^2+4(4\sqrt{2}a^2+(4+3\sqrt{2})b^2)\text{Cos}[2A]
$$

$$-(2(-1+\sqrt{2})a^2+\sqrt{2}b^2)\mathrm{Cos}[4A])\mathrm{Cot}[A]^2 - \frac{b^2\mathrm{ArcTan}[\frac{2z}{\sqrt{-b^2(-3+\sqrt{2}\mathrm{Cos}[A]^2+\mathrm{Cos}[2A])}}]}{8\sqrt{2}\sqrt{-b^2(-3+\sqrt{2}\mathrm{Cos}[A]^2+\mathrm{Cos}[2A])}}$$
$$(-2a^2+14\sqrt{2}a^2+16b^2-13\sqrt{2}b^2-4(4\sqrt{2}a^2+(-4+3\sqrt{2})b^2)$$
$$\mathrm{Cos}[2A]+(2(1+\sqrt{2})a^2+\sqrt{2}b^2)\mathrm{Cos}[4A])\mathrm{Cot}[A]^2$$
$$-a^2b\mathrm{ArcTan}[\frac{z\mathrm{Csc}[A]}{b}]\mathrm{Cos}[A]^2\mathrm{Sin}[A])+1024\mathrm{KKKzTan}[A]^2) \qquad (47)$$

Where JJJ and KKK are constants determined through the initial condition that, according to Soare (1967), imposes, at the top of the hyperbolic tower, that the stresses due to the wind incidence are equal to zero (open top of the hyperbolic tower). The calculation of the stresses, for harmonic 2 ($n=2$), in the hyperbolic tower subjected to the wind action is given by:

$$N_{\varphi\,\theta_2} = -\frac{2\ tg^2 A}{(b^2+z^2)}\,F_1 \qquad (48)$$

$$N_{\varphi_2} = \frac{\sqrt{b^2\sin^2 A+z^2}}{a^2\cos A}\left[\frac{p_w\,(b^2+z^2)}{tg^2 A}\,\frac{\sqrt{b^2\sin^2 A+z^2}}{\sin\ A\ \sqrt{b^2+z^2}} - \frac{d\ F_1}{dz}\right] \qquad (49)$$

The expression (47), responsible for the determination of F_1, although relatively long, can be obtained without great difficulties, for example through the use of the software "MATHEMATICA", analyzing the integrals of the expression (46).

For the determination of $N_{\varphi 2}$, expression (49), it is necessary to determine $\frac{d\ F_1}{dz}$. This derivative will also present a fairly long expression, but can be achieved without great difficulties also with the use of the Software "MATHEMATICA".

By imposing, the initial condition that, according to Soare (1967), at the top of the hyperbolic tower, the stresses due to the wind incidence are equal to zero (open upper top of the hyperbolic tower), and the valuesobtained for both constants are:

$$\{\{\mathrm{JJJ} \to (961.8155549462191 + 0.\,i)\mathrm{Pw}\}\} \qquad (50)$$
$$\{\{\mathrm{KKK} \to (6293.19721644889 - 1100.3192364916897 i)\mathrm{Pw}\}\} \qquad (51)$$

5 CASE STUDY APPLICATION

At the level of a numerical structural example, the state of tension in a cooling tower having the form of a hyperboloid of revolution, subjected to the asymmetric wind loading, will be studied, according to Figure (1).

For the case studied, an example contained in Soare (1967) that was solved through finite differences, the Membrane State will be admitted to the tower structure and the harmonics 0, 1, and 2 will be taken after the harmonic expansion.

Numerical data:

a) collar circle radius (or the real half-axis of the hyperbola meridian.) : $a = 12.750\ m$ (height of $z = 0$);

b) radius of the lower parallel circle : $r_{0,l} = 22.320\ m$ (height of $z = +44\ m$); and

c) total height of the hyperbolic tower : $h = 50\ m$, of which $h_u = 6\ m$ above the collar circle, and $h_l = 44\ m$ below him.

At the top, the tower has a stiffening ring whose dimensions are detailed in Figure (1).

The Cartesian equation of the meridian of the hyperbola has the form:

$$\frac{r_0^2}{a^2} - \frac{z^2}{b^2} = 1 \tag{52}$$

Where b represents the imaginary half-axis, which will be determined.

From the condition $r_0 = r_{0,l}$ to $z = z_l$, one has:

$$b = \frac{z_l}{\sqrt{\dfrac{r_{0,l}^2}{a^2} - 1}} \tag{53}$$

At the height z, the radius of the parallel circle is determined by the formula:

$$r_0 = a\sqrt{1 + \left(\frac{z}{b}\right)^2} \tag{54}$$

From the Cartesian equation of the meridian of hyperbola (54), we deduce:

$$tg\ \varphi = \frac{dz}{dr_0} = \pm \frac{b}{a} \frac{r_0}{\sqrt{r_0^2 - a^2}} = \frac{r_0}{z}\ tg^2 A \tag{55}$$

Where $tg\ A = \dfrac{b}{a}$, with A the angle being formed by the asymptotic meridian of the hyperbola and the axis $O\ r_0$.

For the hyperbola segment below the collar circle, the values of z are positive, for the part superior to that, the values of z are negative.

Two important trigonometric data inherent to the tower are given below:

$$\sin \varphi = \frac{1}{\sqrt{1 + \dfrac{z^2}{r_0^2 \, tg^4 A}}} \tag{56}$$

$$\cos \varphi = \frac{1}{\pm \sqrt{1 + \dfrac{r_0^2 \, tg^4 A}{z^2}}} \tag{57}$$

The principal radii of curvature are given by:

$$r_1 = - \frac{a \sin^2 A \cos A}{\left(\cos^2 A - \cos^2 \varphi \right)^{3/2}} = - \frac{z^3}{b^2 \, tg^2 A \cos^3 \varphi} \tag{58}$$

$$r_2 = \frac{r_0}{\sin \varphi} \tag{59}$$

Assuming that the pressure variation due to wind is sinusoidal along the meridian of the hyperbola, the applied load is given according to equation (19).

The system of differential equations for which an analytical solution is sought is represented by the formulation (24), through which, we arrive at the equation (25) that is responsible for the study involving the analysis of stresses caused by the action of the wind in the cooling tower, shaped like a hyperboloid of revolution:

6 EXPOSURE OF RESULTS AND FINAL CONSIDERATIONS

In the sequence, the results obtained for the three harmonics (0, 1 and 2) responsible for the consideration of the "Membrane State" are presented and commented on in a hyperbolic tower requested by asymmetric loading from the wind.

6.1 RESULTS OBTAINED FOR THE HARMONIC "0" ($n = 0$)

As for harmonic 0, $n = 0$, corresponding to the symmetric state of stress, equations (24) can be solved by direct integration, the results obtained for this harmonic are exact. However, these results were nevertheless compared to those obtained using the "MATHEMATICA" Software, as shown in Table (1) below (where z is the height of the tower in meters).

z [m]	$\frac{N_{\varphi\, OBTIDO}}{p_w}$	$\frac{N_{\varphi\, MATHEMATICA}}{p_w}$	$\lvert \%\ Error \rvert$
- 6	0	0	0
- 3	0.182464	0.182464	0
0	0.244347	0.244347	0

3	0.182464	0.182464	0
6	0	0	0
9	- 0.294041	- 0.294042	0.00034009
12	- 0.68632	- 0.686321	0.0001457
15	- 1.16112	- 1.16112	0
18	- 1.70225	- 1.70225	0
21	- 2.29455	- 2.29455	0
24	- 2.92482	- 2.92482	0
27	- 3.58213	- 3.58213	0
30	- 4.25787	- 4.25787	0
33	- 4.94546	- 4.94545	0.00020221
36	- 5.64002	- 5.64001	0.0001773
39	- 6.33803	- 6.33802	0.00015778
42	- 7.03702	- 7.03701	0.00014211
44	- 7.50269	- 7.50267	0.00026657

Table (1): Variation of $N_{\varphi 0}$ along the height of the tower (for harmonic 0, $n = 0$).

6.2 RESULTS OBTAINED FOR THE HARMONIC "1" ($n = 1$)

For harmonic 1, $n = 1$, the results obtained are also accurate. In the same way as the harmonic 0 study, comparisons were made with the results obtained using the "MATHEMATICA" Software and with the results available in Soare (1967) obtained by Finite Differences, as shown in Table (2) and Table (3).

z $[m]$	$\frac{N_{\varphi\,OBTIDO}}{p_w}$	$\frac{N_{\varphi\,MATHEMATICA}}{p_w}$	$\lvert \%\ Error \rvert$	$\frac{N_{\varphi\,SOARE\ (1967)}}{p_w}$
- 6	0	0	0	0
- 3	0.537138	0.53755	0.076644030	0.5385
0	1.66904	1.66961	0.034139710	1.6620
3	3.35085	3.35133	0.014322670	3.2314
6	5.48437	5.4845	0.002370320	5.2501
9	7.9360	7.93553	0.005922380	7.6833
12	10.5612	10.5598	0.013256070	10.2948
15	13.2271	13.2245	0.019656610	12.9519
18	15.8278	15.8237	0.025903790	15.5549
21	18.2897	18.2837	0.032805350	18.0127
24	20.5703	20.5619	0.040835570	20.3105
27	22.6516	22.6404	0.049444630	22.3984

30	24.5338	24.5193	0.059102140	24.2805
33	26.2281	26.2098	0.069772500	25.9872
36	27.7520	27.7292	0.082156240	27.5197
39	29.1254	29.0974	0.096136020	28.9017
42	30.3685	30.3346	0.111628830	30.1419
44	31.1343	31.0961	0.122694260	31.5632

Table (2): Variation of N_{φ_1} along the height of the tower (for harmonic 1, $n=1$).

z $[m]$	$\frac{N_{\varphi\theta\,OBTIDO}}{p_w}$	$\frac{N_{\varphi\theta\,MATHEMATICA}}{p_w}$	$\lvert \%\ Error \rvert$	$\frac{N_{\varphi\theta\,SOARE\ (1967)}}{p_w}$
- 6	0	0	0	0
- 3	- 3.03504	- 3.03503	0.000329480	- 3.0287
0	- 6.03164	- 6.03161	0.000497380	- 6.0189
3	- 8.85663	- 8.85658	0.000564550	- 8.8374
6	- 11.4005	- 11.4004	0.000877150	- 11.3824
9	- 13.5982	- 13.5982	0	- 13.5806
12	- 15.4353	- 15.4353	0	- 15.4177
15	- 16.9395	- 16.9395	0	- 16.9213
18	- 18.1645	- 18.1646	0.000550520	- 18.1456
21	- 19.1738	- 19.174	0.001043080	- 19.1541
24	- 20.0285	- 20.0289	0.001997110	- 20.0081
27	- 20.7809	- 20.7814	0.002406000	- 20.7599
30	- 21.4720	- 21.4727	0.003259950	- 21.4506
33	- 22.1322	- 22.1329	0.003162710	- 22.1105
36	- 22.7825	- 22.7835	0.004389140	- 22.7607
39	- 23.4373	- 23.4383	0.004266520	- 23.4154
42	- 24.1053	- 24.1066	0.005392710	- 24.0835
44	- 24.5608	- 24.5622	0.005699820	- 24.5401

Table (3): Variation of $N_{\varphi\theta_1}$ along the height of the tower (for harmonic 1, $n=1$).

6.3 RESULTS OBTAINED FOR THE HARMONIC "2" ($n=2$)

For harmonic 2, $n = 2$,the results are obtained by employing the Heron Formula to approximate the square root of an integer that is not a perfect square. Heron's formula made it possible to construct an analytical solution for harmonic 2, that is, through the use of this formula, it was possible to construct a closed analytic expression for the stress functions responsible for the hyperbolic tower wind stresses. And, as was done in the studies involving harmonics 0 and 1, comparisons were made with the results obtained through the use of the "MATHEMATICA" Software, which was used to solve the system of differential equations governing the problem, and with the results available in Soare (1967) obtained by Finite Differences, as shown in Table (4) and Table (5).

z $[m]$	$\frac{N_{\varphi\,OBTIDO}}{p_w}$	$\frac{N_{\varphi\,MATHEMATICA}}{p_w}$	$\lvert \%\ \ Error \rvert$	$\frac{N_{\varphi\,SOARE\ (1967)}}{p_w}$
- 6	0	0	0	0
- 3	1.5994	1.59952	0.0075022507	1.6136
0	5.89152	5.8915	0.000339471	5.8832
3	12.5873	12.5874	0.0007944452	12.4371
6	21.1181	21.1180	0.0004735274	20.8210
9	30.7629	30.7626	0.0009752006	30.4093
12	40.8059	40.8052	0.0017154382	40.4227
15	50.6608	50.6596	0.0023686953	50.2697
18	59.9317	59.9300	0.0028365623	59.5742
21	68.4122	68.4098	0.0035081462	68.0559
24	76.0468	76.0438	0.0039449392	75.7672
27	82.8802	82.8764	0.004584931	82.6246
30	89.0114	89.0066	0.0053925677	88.7561
33	94.5601	94.5542	0.0062394181	94.3617
36	99.6455	99.6385	0.0070249033	99.4639
39	104.3750	104.3670	0.0076646707	104.2145
42	108.8410	108.8320	0.0082689428	108.6537
44	111.7100	111.6990	0.0098469251	112.2108

Table (4): Variation of N_{φ_2} along the height of the tower (for harmonic 2, $n = 2$).

z $[\ m\ ]$	$\frac{N_{\varphi\,\theta\,OBTIDO}}{p_w}$	$\frac{N_{\varphi\,\theta\,MATHEMATICA}}{p_w}$	$\lvert\ \%\ \ Error\ \rvert$	$\frac{N_{\varphi\,\theta\,SOARE\ (1967)}}{p_w}$
- 6	0	0	0	0
- 3	- 6.04195	- 6.04177	0.0029791706	- 5.8895
0	- 11.8337	-11.8332	0.0042252212	- 11.8634
3	- 16.9544	- 16.9538	0.0035389044	- 16.9926
6	- 21.1067	- 21.1060	0.0033164824	- 21.1587
9	- 24.1859	- 24.1851	0.0033077123	- 24.2408
12	- 26.2781	- 26.2772	0.0034249051	- 26.3269
15	- 27.5972	- 27.5962	0.003623556	- 27.6345
18	- 28.4042	- 28.4032	0.0035206061	- 28.4278
21	- 28.9421	- 28.9412	0.0031096569	- 28.9523
24	- 29.4014	- 29.4005	0.0030610787	- 29.3998
27	- 29.9118	- 29.9109	0.003008846	- 29.9004
30	- 30.5495	- 30.5487	0.0026187008	- 30.5305
33	- 31.3511	- 31.3504	0.0022327765	- 31.3262
36	- 32.3264	- 32.3257	0.0021654128	- 32.2972
39	- 33.4694	- 33.4687	0.0020914626	- 33.4369
42	- 34.7656	- 34.7649	0.0020134846	- 34.7307
44	- 35.7061	- 35.7055	0.0016803851	- 35.6724

Table (5): Variation of $N_{\varphi\,\theta_2}$ along the height of the tower (for the harmonic 2, $n = 2$).

6.4 GENERAL RESULTS

It is interesting to note that in the study involving the three harmonics, we used:

$$\begin{cases} b \ = \ 30{,}622 \ \ m \\ \cos \ A \ = \ 0{,}3843 \\ \sin \ A \ = \ 0{,}923 \\ tg \ A \ = \ 2{,}40176 \\ p_w \ = \ cons\tan te \end{cases} \tag{60}$$

And,

$$\begin{cases} N_{\varphi\,\theta\,2}\left(-6\right) = 0 \\ N_{\varphi\,2}\left(-6\right) = 0 \end{cases} \tag{61}$$

According to Soare (1967), N_θ, for the three harmonics studied, it can be obtained as follows:

$$N_{\theta\,n} = -Z_n\,r_2 - N_{\varphi\,n}\,\frac{r_2}{r_1} \Rightarrow N_{\theta\,n} = -p_w\,\sin\varphi\;r_2 - N_{\varphi\,n}\,\frac{r_2}{r_1} \tag{62}$$

In addition, as reported by Soare (1967), the total stresses on the hyperbolic tower subjected to the asymmetric wind action, assuming for the structure of the tower the Membrane State, are given by summing the results obtained for the three harmonics ($n = 0$, $n = 1$, and $n = 2$):

$$\begin{cases} N_{\varphi\,TOTAL} = N_{\varphi\,0} + N_{\varphi\,1} + N_{\varphi\,2} \\ N_{\varphi\,\theta\,TOTAL} = N_{\varphi\,\theta\,0} + N_{\varphi\,\theta\,1} + N_{\varphi\,\theta\,2} \\ N_{\theta\,TOTAL} = N_{\theta\,0} + N_{\theta\,1} + N_{\theta\,2} \end{cases} \tag{63}$$

6.5 FINAL CONSIDERATIONS

The analytical study involving the analysis of the stresses in a hyperbolic tower submitted to the wind, assuming for the structure the State of Membrane, was absolutely accurate when compared to the results obtained through the use of the "MATHEMATICA" Software, which was used to solve the system of differential equations governing the problem involving the asymmetric action of wind in hyperbolic towers.

It is important to note that for harmonics 0 and 1, the results achieved are accurate, since they were obtained from direct integration. For harmonic 2, the Heron formula was used to approximate the square root of an integer that is not a perfect square. The use of this formula provided total subsidies to obtain a complete analytical expression for the stresses that govern the hyperbolic cooling tower, assuming Membrane State for the structure requested by the asymmetric wind loading action.

The formulation used as a base, that of Soare (1967), despite having been published more than fifty years ago, presents itself in very current conditions since it allows that the wind pressure is considered totally in function of the criteria and parameters established by the designer structural; for example, taking into account the "Group Effect" for the structure, in addition to the latest procedures from the last studies involving asymmetric loads, such as wind.

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